Deflection of Beams with Unequal Overhangs

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# Introduction

This analysis seeks to determine closed form equations for the rotation (slope tangent to the deflection curve) and the deflection of a prismatic, uniformly load, simple span beam with unequal overhangs.

# Development of Exact Solution

The geometry, coordinate systems, and shear and moment diagrams for a uniformly loaded beam with equal overhangs is shown below.



The following relationships are known:

The following calculations will yield the deflection of a beam with unequal overhangs.

1. Establish relationships for shear,
2. Establish relationships for moment,
3. Establish relationships for rotation,
4. Establish relationships for deflection,

# Establish relationships for shear

The relationships for shear are obvious from the shear diagram

# Establish relationships for moment

## Moment

Solve for the constant of integration using the boundary condition

Substitute

## Moment

Solve for the constant of integration using the boundary condition

Substitute

Simplify

## Moment

Solve for the constant of integration using the boundary condition

Substitute

Simplify

# Establish relationships for rotation

## Rotation

The constant of integration will be solved later

## Rotation

The constant of integration will be solved later

## Rotation

The constant of integration will be solved later

# Establish relationships for deflection

## Deflection

Solve for the constant of integration using the boundary condition

Substitute

Simplify

## Deflection

Solve for the constant of integration using the boundary condition

Substitute

## Deflection

Solve for the constant of integration using the boundary condition

## Solve for remaining constants of integration

The rotations on both sides of support A are equal to one another.

The rotations on both sides of support B are equal to one another.

Re-arrange

Expand

Simplify

The deflection at support B is 0. Solve for

Re-arrange

Expand and simplify

Solve for

Substitute

Expand and simplify

Solve for

Substitute

Expand and simplify

## Finalize rotation equations

Substitute the constants of integration into the rotation equations

## Rotation

Substitute

Simplify

## Rotation

Substitute

Simplify

## Rotation

Substitute

Simplify

## Finalize deflection equations

Substitute the constants of integration in to the deflection equations

## Deflection

Substitute

Expand and simplify

## Deflection

Substitute

Expand and simplify

## Deflection

Substitute

Expand and simplify

# Maximum moment

Moment is maximized when shear is zero. Between

Solve for x.

Evaluate moment at x.

If

when

# Maximum deflection

Deflection is maximized when rotation is zero. Between

Solve for x.

There isn’t any value in solving this equation for x at this time. Return to this effort later. Once there is a solution for x, substitute it into and simplify to have the maximum deflection.